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VULNERABILITY OF STRIKE FORCES TO SUBMARINE ATTACK

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20 November 1963

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A Project Strike Document

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By C.E. Behrens

Research Contribution No. 32

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Research Contribution

NAVAL WARFARE ANALYSIS GROUP

Center for Naval Analyses

WASHINGTON 25, D. C.

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RESEARCH CONTRIBUTION

Naval Warfare Analysis Group

CENTER FOR NAVAL ANALYSES

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ABSTRACT

An examination is made of the vulnerability of carriers to attacks by submarines using torpedoes or short-range missiles. A mathematical model is derived, and the interactions of several parameters are examined.

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FOREWORD

This research contribution is one of a series of papers resulting from the many and diverse research efforts undertaken as parts of CNA Project Strike, the general objective of which is to determine what kinds of strike forces the Navy should plan to have and operate during limited wars in the 1970's. Issued in advance of publication of the formal report on Phase I of Strike, these papers do not represent the official opinion either of CNA or of the Navy, and should not be construed to contain either conclusions or recommendations that will necessarily appear in the formal report.

INTRODUCTION

This research contribution has two parts. The first is the derivation of a mathematical model for estimating the vulnerability of carriers to attacks by submarines using torpedoes or short-range missiles. The second part presents the results of computations based on the model. These results are in parametric form and may be applied generally to any situation involving attacks with short-range weapons.

THE MODEL

Measure of Effectiveness

The measure of effectiveness used here is the probability, $P_s(t)$, that the carrier will survive submarine attacks for a time t after it enters marine-infested region. It is given by the following expression:

$$P_s(t) = \exp\left[-\left(\frac{Wvfg}{A}\right)\left(\frac{n_o}{\lambda}\right)\left(1 - e^{-\lambda t}\right)\right] \tag{1}$$

where

- W = attack width of the submarine;

  $= 2r \frac{u}{v} \quad (u < v)$

  $= 2r \quad (u \geq v)$

- r = range at which submarine detects the carrier;
- u = submarine SOA during approach;
- v = strike force SOA;
- f = probability that the submarine makes an undetected approach;
- g = conditional probability that the submarine launches a successful attack (severely damaging or sinking the carrier);
- A = area of the region of operations;
- n_o = number of submarines in the area at the beginning of the combat;
- λ = if more submarines enter the area the treatment page 3 would apply; expected submarine loss per unit time

  $= \frac{(S)(h)}{A}$;

- S = combined search rate of the ASW forces;
- h = conditional probability that the submarine is killed after being contacted.

Derivation

Equation (1) is derived as follows:

If $P_s(t)$ is the probability that a carrier survives until time t when operating in an area in which q is the probability of its being killed (sunk or severely damaged) per unit time, the probability that the carrier survives until time t + dt is then the product of the probability that it survives until time t and the probability that it is not killed during the interval dt, i.e.,

$$P_s(t + dt) = (1 - q\,dt)\;P_s(t) \;.$$

The solution to this equation is

$$P_s(t) = \exp\left[-\int_0^t q(t)\,dt\right] \tag{2}$$

under the boundary condition $P_s(0) = 1$.

The probability q(t) is proportional to the submarine density, n(t)/A. Since number of submarines, n(t), is a function of time, q(t) is also a function of time. It is calculated as follows:

The carrier generates an area per unit time, Wv, within which a submarine making contact can attack. The probability per unit time, q(t) that a submarine will close and carry out a successful attack is therefore

$$q(t) = \frac{Wvfgn(t)}{A} \;. \tag{3}$$

The number of submarines in the area A decreases exponentially with time:

$$n(t) = n_0 e^{-\lambda t} \;. \tag{4}$$

Substituting equations (3) and (4) in equation (2), we have

$$P_s(t) = \exp\left[-\int_0^t \left(\frac{Wvfg}{A}\right) n_0 e^{-\lambda t}\,dt\right] \tag{5}$$

which, upon integrating, yields equation (1).

Equation (1) assumes that n_o is constant; i.e., the enemy has all his available submarines deployed in area A at the beginning of the operation. The situation in which submarines enter the area A during the operation, i.e., $n_o = \Phi(t)$, is of interest. It may be handled in two ways:

(1) At $t = 0$, there are n_o submarines in the area. At $t = \tau_i$ $(i = 1, 2, 3 \ldots.)$, ν_i $(i = 1, 2, 3, \ldots.)$ additional submarines enter the area. For each τ_i, $n_i(t)$ is computed from equation (4); the corresponding ν_i is added to $n_i(t)$, and equation (1) applied iteratively.

(2) At $t = 0$, $n = n_o$. Submarines enter the area continuously at an average rate of k per unit time. In this case, the value of n(t) may be derived as follows:

$$\frac{dn(t)}{dt} = -\lambda n(t) + k \quad . \tag{6}$$

The solution of this equation is

$$n(t) = n_o e^{-\lambda t} + \frac{k}{\lambda}\left(1 - e^{-\lambda t}\right) \quad . \tag{7}$$

Substituting equations (3) and (7) into equation (2), we have, after integrating,

$$P_s(t) = \exp\left[-\frac{Wvfg}{A}\left\{\left(\frac{n_o}{\lambda} - \frac{k}{\lambda^2}\right)\left(1 - e^{-\lambda t}\right) + \frac{kt}{\lambda}\right\}\right] \quad . \tag{8}$$

PARAMETRIC ANALYSIS

The purpose of these calculations is to provide estimates of the vulnerability of carriers to threats of potential magnitude, given certain degrees of ASW effectiveness. They yield some insight into the relative sensitivity of the vulnerability to parameter values. Since the treatment is perfectly general no definite conclusions are stated.

The measure of effectiveness selected for the analysis is the probability, $P_s(t)$, that the carrier will survive the attack for a time t after the strike force enters the area of submarine operations. Assume that there are initially n_o submarines uniformly distributed in an area of A sq. mi. through which the strike force must transit or in which it must operate, and that these submarines are subjected to attrition of λ per unit time. Now let us rewrite equation (5) above as follows:

$$P_s(t) = \exp\left[-\int_o^t \left(\frac{Wvfg}{A}\right) n_o e^{-\lambda t} dt\right] = \exp\left[-Q_o \int_o^t e^{-\lambda t} dt\right]$$

where

Q_o = the probability per unit time that the carrier will be sunk (or severely damaged) if there is no attrition of submarines

and

$-\int_o^t e^{-\lambda t} dt$ = the attrition factor.

Q_o measures the average severity of the threat during an operation, and for a given set of parameter values may be treated as a constant. It is the product of the probability per unit time, q_a, that a submarine can close the task force to attack position, and the conditional probability, q_k, that it will make a successful attack. The probability q_a is given by $Wv\left(\frac{n_o}{A}\right)$, where

W = attack width of the submarine

$$= 2r \frac{u}{v} \quad (u < v)$$

$$2r \quad (u > v)$$

r = range at which the submarine detects the carrier;

u = average SOA of the submarine during its approach;

v = average SOA of the strike force.

The probability q_k is the probability that a submarine which can close to attack range makes a successful attack; it is given by the product of f and g, where

f = probability that the submarine makes an undetected approach to attack position;

g = probability that the weapons launched by the submarine inflict the prescribed damage.

Figure 1 shows representative values of q_a for several values of u, v, and r, as a function of submarine density, n_o/A. These are combined with arbitrary values of q_k to yield Q_o. The three values of q_k shown in figure 1 might be achieved under the following conditions:

(a) a fairly tight screen yielding f = 0.5 at maximum weapon range, with g = 0.25; $q_k = 0.125$;

(b) an ineffective screen, with f = 0.8, and g = 0.6; $q_k = 0.5$;

(c) zero screen effectiveness (f = 1.0) and extremely lethal submarine weapons (g = 0.9); e.g., ASTOR type torpedoes with nuclear warheads; $q_k = 0.9$.

The attrition rate λ, is given by

$$\lambda = \frac{Sh}{A}$$

where

S = combined hourly search rate of the antisubmarine forces in the area A

and

h = conditional probability that a detected submarine will be killed.

The search rate, S, and hence the attrition rate, λ, are assumed to be constant, on the average, during the operation. This assumption neglects the possible attrition of the ASW search forces by enemy action, and also the case in which, because of evidence of submarine concentration, the search effort may be intensified temporarily by increasing aircraft activity beyond normal usage rates. Any likely variations in the search effort, however, will not significantly affect the value of $P_s(t)$; figure 3 shows that changing the value of λ by as much as a factor of 3 to 5, in the range $0.01 < \lambda < 0.1$, results in a difference of the order of only ten percent in the survival probability during the first day of combat.

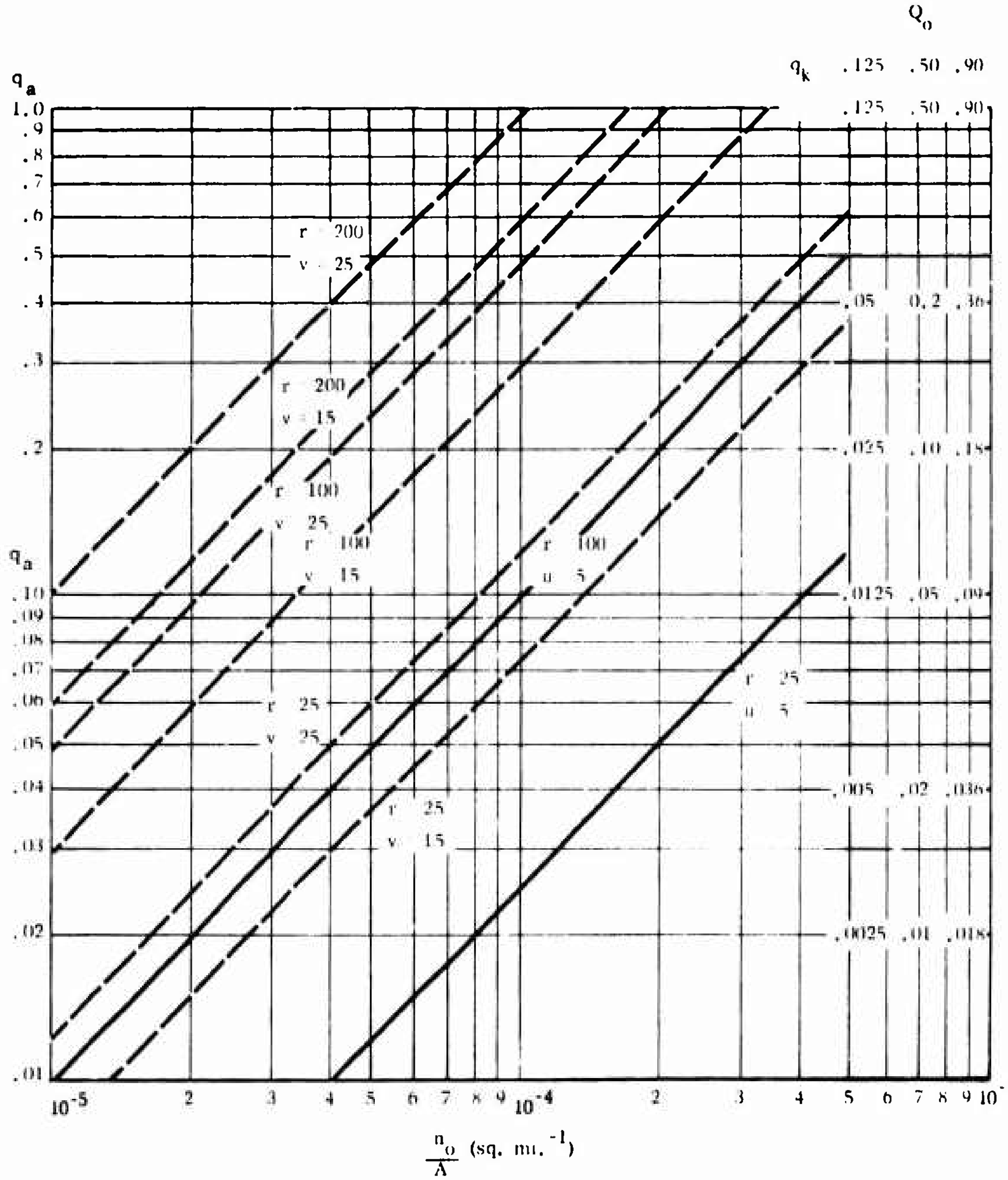

FIG. 1: PROBABILITY PER HOUR, Q_o, THAT CARRIER WILL BE SUCCESSFULLY ATTACKED

(no attrition of submarines)

Graphs of the $P_s(t)$ function are shown in figure 2 for several values of Q_0, and for two attrition rates: a high one ($\lambda = 0.1$) and a low one ($\lambda = 0.01$). The value $Q_0 = 0.05$ can be used as a rough measure of a severe threat, since it results in survival probabilities of 0.6 or less, even with heavy attrition of submarines. As is to be expected if one assumes a constant average attrition rate, the vulnerability is greatest during the first day; for a high attrition rate ($\lambda = 0.1$) it is actually confined to the first 24 hours, and most of the damage is done in the first 12 hours. This is shown in figures 3 and 4, in which $P_s(t)$, for 24 hours and 12 hours, respectively, is plotted as a function of Q_0.

To ensure survivability of the order of 0.8 it is seen from figure 3 that the threat must be kept to a severity indicated by $Q_0 < 0.03$, unless the operating area can be saturated with ASW forces, such that they can sweep out the whole region every hour or so ($\lambda = 0.5$). Neglecting attrition for the moment, Q_0 can be reduced by reducing

(a) the initial submarine density, $\frac{n_0}{A}$;

(b) the submarine 's detection range, r;

(c) the probability of undetected approach, f;

(d) the kill probability, g.

The initial submarine density may be reduced by

(a) sterilizing the area before the strike force enters it;

(b) forcing the submarines to cover a larger area; e.g., by deceptive measures or extended formations.

Reducing the submarine's detection range could be accomplished by quieting the ships of the strike force. It might also be practicable to mask the sound output of the strike force by detonating explosives in the vicinity.

The probability that the submarine can make an undetected approach to attack position depends on the effectiveness of the screen. Both the screen effectiveness, as measured by the probability of detecting an intruder, and the attrition rate, are directly proportional to the search rate.

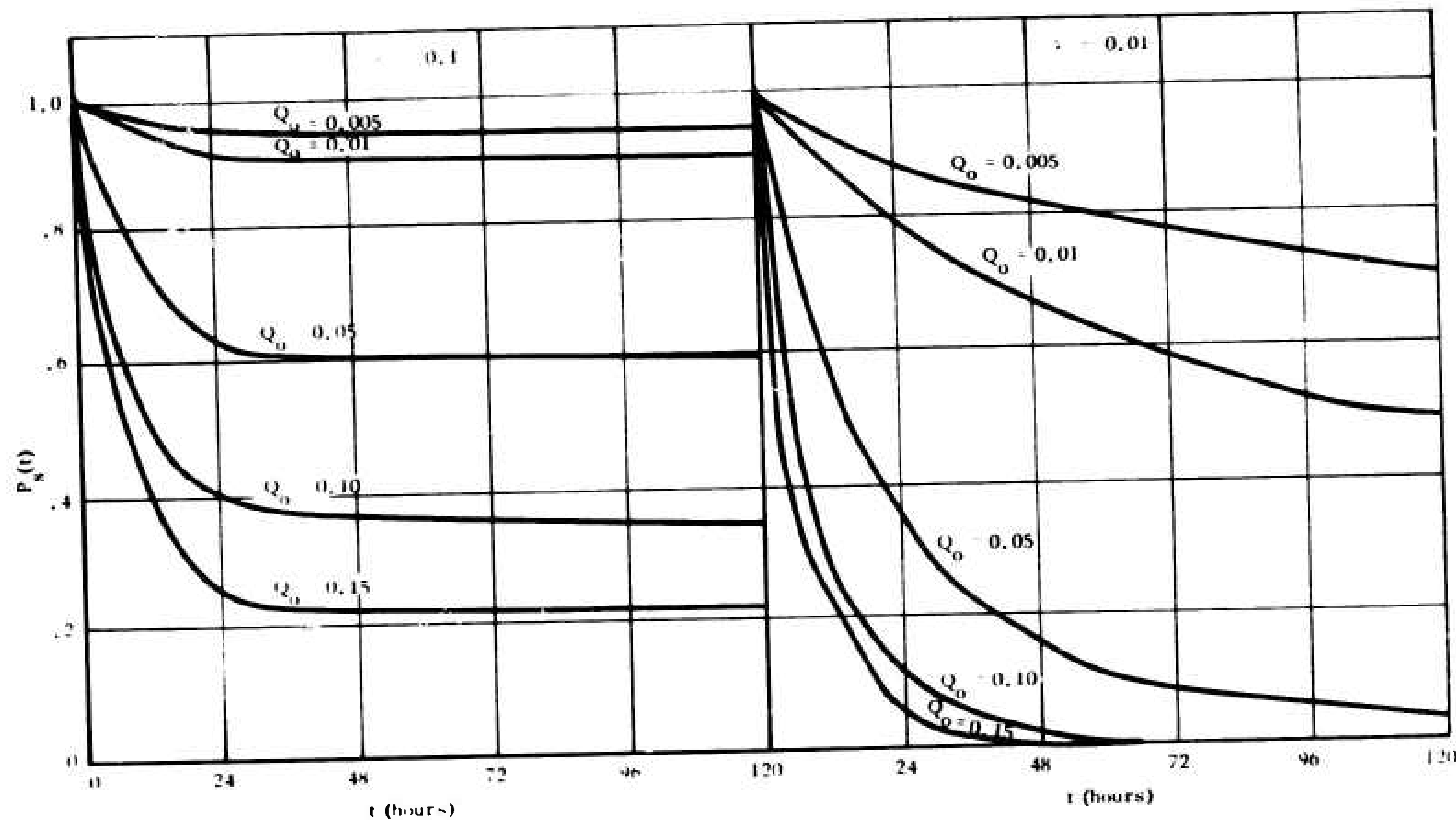

FIG. 2: PROBABILITY OF SURVIVAL ($P_s(t)$) OF CARRIER

(λ = expected submarine loss per hour)

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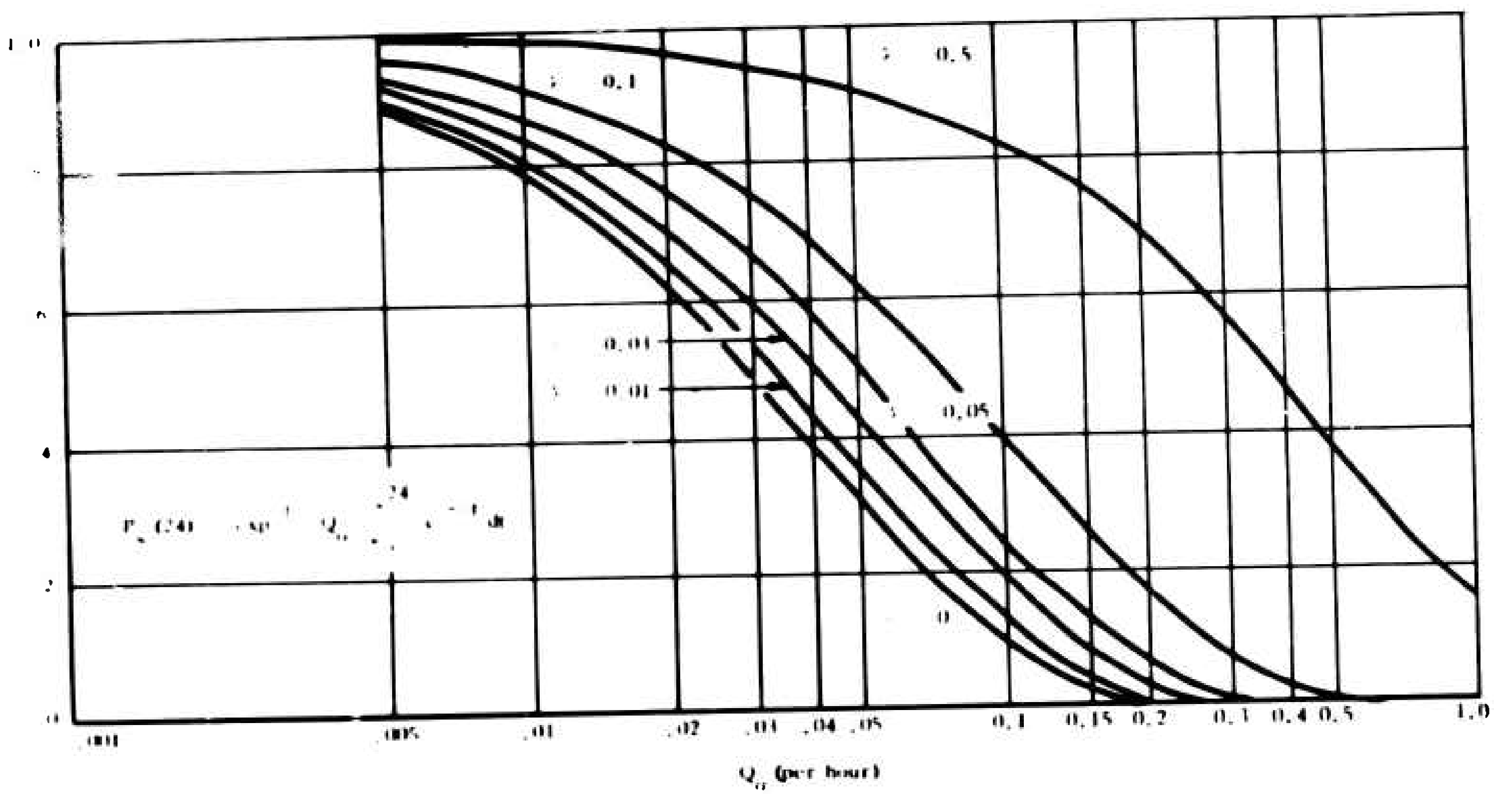

FIG. 3: **PROBABILITY OF SURVIVAL OF CARRIER AFTER 24 HOURS**

(expected submarine loss per hour)

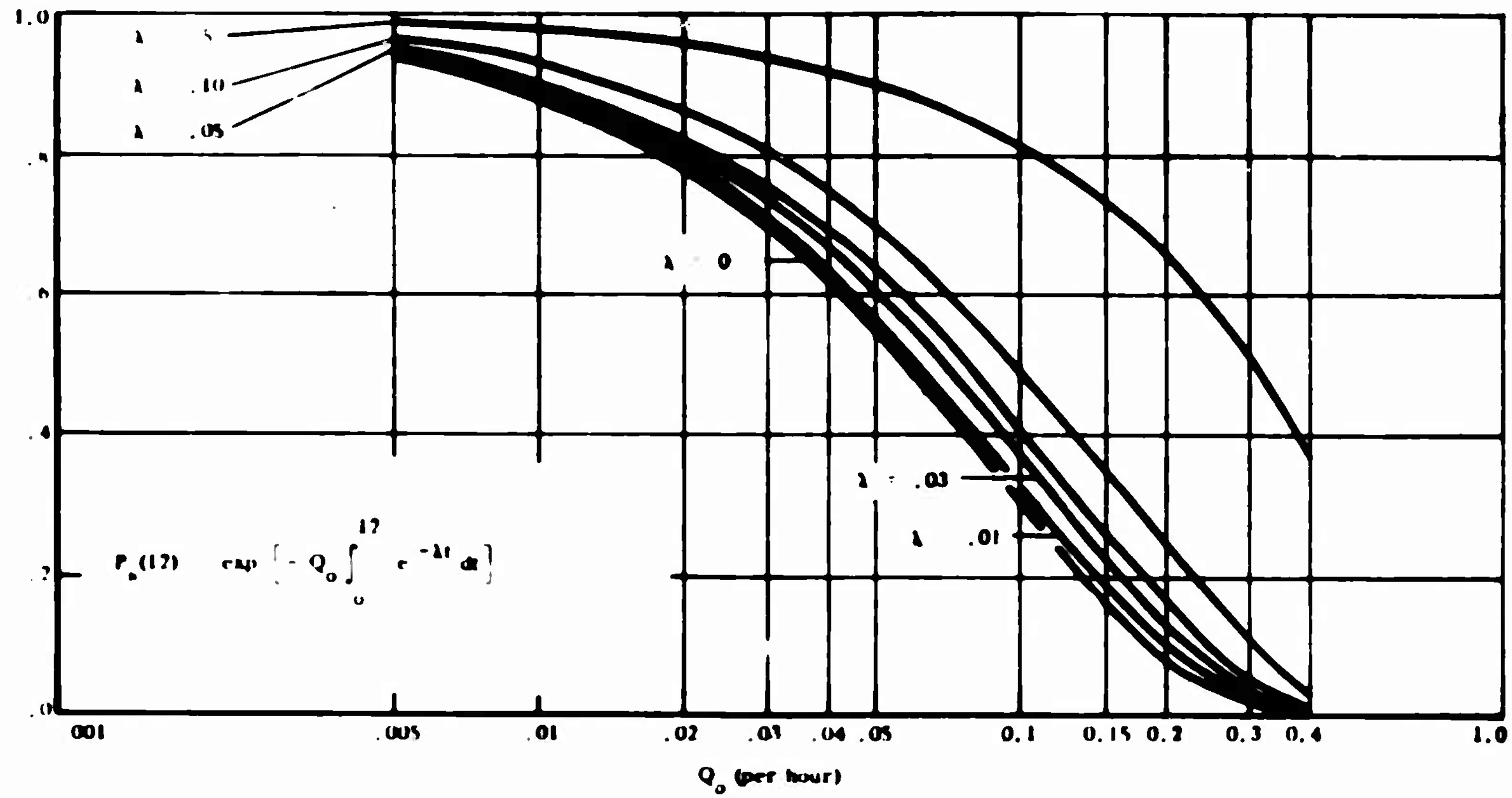

FIG. 4: PROBABILITY OF SURVIVAL OF CARRIER AFTER 12 HOURS
(expected submarine loss per hour)

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